

## Generalized symmetry breaking on orbifolds

Jonathan A. Bagger,<sup>1</sup> Ferruccio Ferruglio<sup>2</sup> and Fabio Zwirner<sup>3</sup>

<sup>1</sup> *Department of Physics and Astronomy, Johns Hopkins University,  
3400 North Charles Street, Baltimore, MD, 21218, USA*

<sup>2</sup> *Dipartimento di Fisica ‘G. Galilei’, Università di Padova and INFN,  
Sezione di Padova, Via Marzolo 8, I-35131 Padova, Italy*

<sup>3</sup> *Dipartimento di Fisica, Università di Roma ‘La Sapienza’ and INFN,  
Sezione di Roma, P.le Aldo Moro 2, I-00185 Rome, Italy  
(Dated: July 16, 2001)*

We reconsider the phenomenon of mass generation via coordinate-dependent compactifications of higher-dimensional theories on orbifolds. For definiteness, we study a generic five-dimensional (5D) theory compactified on  $S^1/Z_2$ . We show that the presence of fixed points, where the fields or their derivatives may be discontinuous, permits new realizations of the Scherk-Schwarz mechanism where, for example, the mass terms are localized at the orbifold fixed points. Our technique can be used to describe the explicit breaking of global flavor symmetries and supersymmetries by brane-localized mass terms. It can also be applied to the spontaneous breaking of local symmetries, such as gauge symmetries or supergravities.

### INTRODUCTION

Coordinate-dependent compactifications of higher-dimensional theories, first proposed by Scherk and Schwarz [1], provide an elegant and efficient mechanism for mass generation and symmetry breaking. The basic idea is very simple: one twists the boundary conditions in the compact extra dimensions by a global symmetry of the action. From a four-dimensional (4D) point of view, this twist induces mass terms that break the symmetries with which it does not commute. (For early applications, see [2].)

In this letter we study Scherk-Schwarz compactifications of field theories on orbifolds. We restrict our attention to compactifications from five to four dimensions on the orbifold  $S^1/Z_2$ . Consistent Scherk-Schwarz compactifications on this space were first formulated in string theory [3] and later in field theory [4]. Related phenomenology was explored in [5], related field-theoretical models in [6], and more string realizations in [7].

In what follows we present a new type of coordinate-dependent compactification in which the fields and their derivatives can jump at the orbifold fixed points. The discontinuities give rise to new possibilities for symmetry breaking. In particular, they give rise to mass terms that can be localized, partially or even completely, at the orbifold fixed points. This suggests a close connection between our realization and localized brane dynamics. Moreover, in contrast to the standard case where the Scherk-Schwarz mass spectrum is completely determined by the overall twist, we will find that the spectrum also depends on the behavior of the fields at the orbifold fixed points.

Our results have a wide range of applications. They can be used to generate the explicit breaking of global symmetries, such as rigid supersymmetry or flavor symmetry. They can also be used to induce the spontaneous

breaking of local symmetries, such as grand unified gauge symmetries or supergravity. Indeed, as we discuss in a companion paper [8], our results encompass such dynamical supersymmetry breaking mechanisms as gaugino condensation at the orbifold fixed points [9].

The plan of this paper is as follows. We first explain the general features of our construction. We then illustrate our results with a simple example, a free 5D massless fermion with  $U(1)$  twisted boundary conditions. We conclude with some comments on the spontaneous breaking of local symmetries, in particular supergravity, and on further applications.

### GENERAL MECHANISM

We consider a generic 5D theory compactified on the orbifold  $S^1/Z_2$ , with space-time coordinates  $x^M \equiv (x^m, y)$ . We work on the covering space  $S^1$ , defined by identifying the coordinates  $y$  and  $y + 2\pi R$ , where  $R$  is the radius. We project to the orbifold  $S^1/Z_2$  by further identifying the coordinates  $y$  and  $-y$ . We denote by  $\Psi(x^m, y)$  all the fields of the 5D theory, classifying them in representations of the 4D Lorentz group.

We assume that the theory has a continuous global symmetry, whose action on the fields is given by  $\Psi \rightarrow \Psi' = U\Psi$ , where  $U$  is a unitary matrix. We define the  $Z_2$  transformations of the fields by

$$\Psi(-y) = Z\Psi(y), \quad (1)$$

where  $Z$  is a matrix such that  $Z^2 = 1$ . It is not restrictive for us to take a basis in which  $Z$  is diagonal,

$$\Psi = \begin{pmatrix} \Psi^+ \\ \Psi^- \end{pmatrix}, \quad Z = \text{diag}(1, \dots, 1, -1, \dots, -1). \quad (2)$$

We implement the Scherk-Schwarz mechanism by twisting the boundary conditions on  $S^1$ . Since the fields

$\Psi(y)$  are multi-valued on the circle, it is convenient to define the twist on the real axis:

$$\Psi(y) = U_{\vec{\beta}} \Psi(y + 2\pi R), \quad (3)$$

where the matrix  $U_{\vec{\beta}}$  depends on the real parameters  $\vec{\beta}$ , but not on the space-time coordinates. A well-known consistency condition [3, 4] between the twist and the orbifold projection is that

$$U_{\vec{\beta}} Z U_{\vec{\beta}} = Z. \quad (4)$$

If we write  $U_{\vec{\beta}} = \exp(i\vec{\beta} \cdot \vec{T})$ , where the matrix  $\vec{\beta} \cdot \vec{T}$  is hermitian, we see that eq. (4) is satisfied if  $\{\vec{\beta} \cdot \vec{T}, Z\} = 0$ . This implies that the generator  $\vec{\beta} \cdot \vec{T}$  is purely off-diagonal in the basis of eq. (2).

Our theory is defined on the orbifold  $S^1/Z_2$ , so we allow the fields to jump at the orbifold fixed points:

$$\Psi(y_q + \xi) = U_q \Psi(y_q - \xi), \quad (5)$$

where  $y_q = q\pi R$ ,  $q \in Z$ ,  $0 < \xi \ll 1$  and  $U_q$  is a global symmetry transformation. The jumps across points related by a  $2\pi R$  translation must be the same, so

$$U_{2q} \equiv U_0, \quad U_{2q+1} \equiv U_\pi. \quad (6)$$

A consistency condition identical to (4) holds for each of the jumps:

$$U_q Z U_q = Z. \quad (7)$$

The physical spectrum is controlled by the Scherk-Schwarz twist and by the jumps at the orbifold fixed points. The discontinuities are the result of mass terms localized at the fixed points. In the next section, we shall see that the mass terms can be described by more than one brane action. We will also see that the theory with discontinuities is equivalent to a conventional Scherk-Schwarz theory with a modified twist. In particular, it is possible for the discontinuities to completely remove the symmetry breaking induced by the twist!

### EXAMPLE

To illustrate our mechanism in a simple setting, we consider the equation of motion for a free 5D massless fermion, written in terms of 5D fields with 4D spinor indices

$$i\sigma^m \partial_m \bar{\Psi} - i\sigma^2 \partial_y \Psi = 0, \quad (8)$$

valid in each region  $y_q < y < y_{q+1}$  of the real axis. In the notation of eqs. (1) and (2), we write:

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \quad \bar{\Psi} = \begin{pmatrix} \bar{\psi}_1 \\ \bar{\psi}_2 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (9)$$

The equation of motion (8) is invariant under global  $SU(2)$  transformations of the form  $\Psi' = U\Psi$ , where  $U \in SU(2)$ . We take

$$U_\beta = \exp(i\beta\sigma^2) = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix}, \quad (10)$$

$$U_q = \exp(i\delta_q\sigma^2) = \begin{pmatrix} \cos\delta_q & \sin\delta_q \\ -\sin\delta_q & \cos\delta_q \end{pmatrix}, \quad (11)$$

where  $\delta_{2q} = \delta_0$  and  $\delta_{2q+1} = \delta_\pi$  for any  $q \in Z$ .

We seek solutions  $\Psi(y)$  to eq. (8), with the boundary conditions of eqs. (10) and (11). Exploiting the fact that  $i\sigma^m \partial_m \bar{\Psi} = m\Psi$ , we find

$$\Psi(y) = \chi \begin{pmatrix} \cos[my - \alpha(y)] \\ \sin[my - \alpha(y)] \end{pmatrix}, \quad (12)$$

where  $\chi$  is a  $y$ -independent 4D spinor,

$$m = \frac{n}{R} - \frac{(\beta - \delta_0 - \delta_\pi)}{2\pi R}, \quad (n \in Z), \quad (13)$$

and

$$\alpha(y) = \frac{\delta_0 - \delta_\pi}{4} \epsilon(y) + \frac{\delta_0 + \delta_\pi}{4} \eta(y). \quad (14)$$

Here  $\epsilon(y)$  is the ‘sign’ function defined on  $S^1$ , and

$$\eta(y) = 2q + 1, \quad y_q < y < y_{q+1}, \quad (q \in Z), \quad (15)$$

is the ‘staircase’ function that steps by two units every  $\pi R$  along  $y$ . The function  $\alpha(y)$  satisfies

$$\alpha(y + 2\pi R) = \alpha(y) + \delta_0 + \delta_\pi. \quad (16)$$

so the solution (12) has the correct Scherk-Schwarz twist. Sample solutions are shown in Fig. 1.

The spectrum (13) is characterized by a uniform shift with respect to a traditional Kaluza-Klein compactification. In contrast to the usual Scherk-Schwarz mechanism, however, the shift depends on the jumps  $\delta_0$  and  $\delta_\pi$ , as well as on the twist  $\beta$ . In particular, it is possible to have a vanishing shift for nonvanishing  $\beta$ . In the limit  $\delta_q \rightarrow 0$ , our results reduce to the conventional Scherk-Schwarz spectrum. Note that the eigenfunction of eq. (12) is discontinuous: the even part has cusps and the odd part has jumps at  $y = y_q$ , as required by the boundary conditions. In the limit  $\delta_q \rightarrow 0$  the eigenfunction becomes regular everywhere.

For any  $\delta_q$ , the system is equivalent to a conventional Scherk-Schwarz compactification with twist  $\beta_c = \beta - \delta_0 - \delta_\pi$ . The new field variable,  $\Psi_c$ , is related to the discontinuous variable,  $\Psi$ , via the generalized function  $\alpha(y)$ ,

$$\begin{pmatrix} \psi_{1c} \\ \psi_{2c} \end{pmatrix} = \begin{pmatrix} \cos\alpha(y) & \sin\alpha(y) \\ -\sin\alpha(y) & \cos\alpha(y) \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}. \quad (17)$$

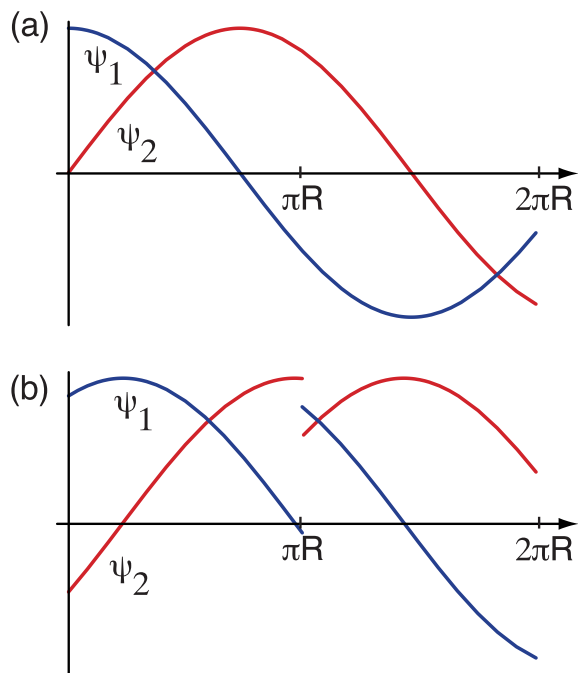


FIG. 1: Sample profiles for the functions  $\psi_1$  and  $\psi_2$ , for two cases with identical spectra. (a) No jumps:  $\beta = \beta_0$ ,  $\delta_0 = \delta_\pi = 0$ . (b) No twist:  $\beta = 0$ ,  $\delta_0 = \delta_\pi = -\beta_0/2$ . In each case, the plot for  $y < 0$  is obtained by reflection, consistent with the parity symmetry.

This is reminiscent of strong CP violation, where the physical order parameter is not  $\theta$ , but the combination  $\theta - \arg \det m_q$ , where  $m_q$  is the quark mass matrix. Similarly, the mass shift of our system is controlled not by  $\beta$  alone, but by the twist  $\beta_c$ , which includes contributions from jumps in the fermion fields. As in QCD, where we can eliminate the phase in  $\det m_q$  by a chiral transformation, here we can remove the jumps by a redefinition of the fermion fields. In the new basis, there are no jumps, but the twist acquires an additional contribution.

Discontinuous field variables arise from mass terms localized at the fixed points. This can be seen by starting with the Lagrangian  $\mathcal{L}$  for the fermions  $\psi_c^{1,2}(y)$ , characterized by a twist  $\beta_c = \beta - \delta_0 - \delta_\pi$ :

$$\mathcal{L}(\psi_c) = i\bar{\psi}_c^1 \bar{\sigma}^m \partial_m \psi_c^1 + i\bar{\psi}_c^2 \bar{\sigma}^m \partial_m \psi_c^2 + \left[ \frac{1}{2} (\psi_c^2 \partial_y \psi_c^1 - \psi_c^1 \partial_y \psi_c^2) + \text{h.c.} \right]. \quad (18)$$

If we perform the field redefinition of eq. (17), the 5D Lagrangian becomes:

$$\mathcal{L}(\psi_c) = \mathcal{L}(\psi) + \mathcal{L}_{brane}(\psi), \quad (19)$$

where

$$\mathcal{L}_{brane}(\psi) = -\frac{1}{2} \alpha'(y) (\psi_1 \psi_1 + \psi_2 \psi_2) + \text{h.c.}, \quad (20)$$

and

$$\alpha'(y) = \sum_{q=-\infty}^{+\infty} [\delta_0 \delta(y - y_{2q}) + \delta_\pi \delta(y - y_{2q+1})]. \quad (21)$$

We see that the jumps  $\delta_q$  arise from fermion mass terms localized at the orbifold fixed points.

The discontinuities of the fields can be recovered by integrating the equations of motion. The trick is to find the correct equations. We avoid all subtleties associated with discontinuous field variables by *defining* the term that appears in the brane action to be *continuous* across the orbifold fixed points. For the case at hand, this means one must choose the field variables so that the combination  $\psi_1 \psi_1 + \psi_2 \psi_2$  is continuous. Alternatively, one can obtain the equations of motion by first regularizing the delta functions, so that  $\psi_1$  and  $\psi_2$  are continuous, and then taking the singular limit.

It is interesting to note that the same physical system can be obtained from another brane Lagrangian, one in which we treat the even field  $\psi_1(y)$  as continuous. The discontinuity of the odd field  $\psi_2(y)$  is then

$$\psi_2(y_q + \xi) - \psi_2(y_q - \xi) = -2 \tan \frac{\delta_q}{2} \psi_1(y_q). \quad (22)$$

This jump is reproduced by the brane Lagrangian

$$\mathcal{L}'_{brane}(\psi) = -\frac{1}{2} f(y) \psi_1 \psi_1 + \text{h.c.}, \quad (23)$$

where

$$f(y) = 2 \sum_{q \in \mathbb{Z}} \left[ \tan \frac{\delta_0}{2} \delta(y - y_{2q}) + \tan \frac{\delta_\pi}{2} \delta(y - y_{2q+1}) \right]. \quad (24)$$

In this case, we vary with respect to  $\psi_1(y)$  and  $\psi_2(y)$ ; the discontinuous field  $\psi_2(y)$  does not appear in the brane Lagrangian.

In summary, the brane Lagrangians (20) and (23) give rise to equivalent theories in the absence of brane interactions, provided we use an appropriate procedure to derive the equations of motion.

## CONCLUSIONS

In this letter we have studied coordinate dependent compactifications of field theories on orbifolds. We have seen that the mass spectrum depends on an overall twist of the fields, together with the jumps of the fields at the orbifold fixed points. Such compactifications can break the symmetries of a theory, either global and local. The order parameter is nonlocal, in the sense that it is determined by a combination of the twist and the discontinuities.

In a supersymmetric Yang-Mills theory, for example, the twist and the jumps are defined by a  $U(1)_R$  subgroup

of  $SU(2)_R$ . From a 4D point of view, this typically breaks the  $N = 1$  supersymmetry that survives the orbifold projection. Note, though, that it is possible for supersymmetry to remain unbroken. For instance, when  $\beta = 0$ , supersymmetry is preserved in the presence of opposite, nonvanishing jumps at  $y = y_{2q}$  and  $y = y_{2q+1}$ , in analogy with a phenomenon first discussed in  $M$ -theory [9]. This example can be readily extended to the case where the fermions  $\psi_1$  and  $\psi_2$  come in  $n$  distinct copies, in which case flavor symmetry is broken if the matrices  $U_\beta$  and  $U_q$  have a non-trivial structure in flavor space.

It is important to note that our mechanism provides a self-consistent way of introducing other interaction terms, such as Yukawa couplings, or even kinetic terms, that are localized at the fixed points. Such terms will always occur in non-renormalizable theories, including supergravity, where the kinetic terms typically have a non-canonical (and non-renormalizable) form.

It would be interesting to find string realizations of our mechanism, which so far are missing. These would give rise to models where mass terms for the untwisted fields are localized at the fixed points of a non-freely acting orbifold.

## ACKNOWLEDGEMENTS

We thank D. Belyaev, K. Dienes, A. Hebecker, J.-P. Hurni, E. Kiritsis, C. Kounnas, A. Masiero, M. Porrati, L. Silvestrini, C. Scrucca, M. Serone and N. Weiner for discussions. We especially thank C. Biggio for her valuable help in improving the first version of the manuscript. We also thank the Aspen Center of Physics, where part of this work was done, for its warm hospitality. J.B. is supported by the U.S. National Science Foundation, grant NSF-PHY-9970781. F.F. and F.Z. are partially supported by the European Program HPRN-CT-2000-00148.

---

[1] J. Scherk and J. H. Schwarz, Phys. Lett. B **82**, 60 (1979) and Nucl. Phys. B **153**, 61 (1979).

[2] P. Fayet, Phys. Lett. B **159**, 121 (1985) and Nucl. Phys. B **263**, 649 (1986);

[3] S. Ferrara, C. Kounnas and M. Porrati, Phys. Lett. B **206**, 25 (1988); C. Kounnas and M. Porrati, Nucl. Phys. B **310**, 355 (1988); S. Ferrara, C. Kounnas, M. Porrati and F. Zwirner, Nucl. Phys. B **318**, 75 (1989).

[4] M. Porrati and F. Zwirner, Nucl. Phys. B **326**, 162 (1989); E. Dudas and C. Grojean, Nucl. Phys. B **507**, 553 (1997); I. Antoniadis and M. Quiros, Nucl. Phys. B **505**, 109 (1997) and Phys. Lett. B **416**, 327 (1998).

[5] I. Antoniadis, Phys. Lett. B **246**, 377 (1990); I. Antoniadis, C. Munoz and M. Quiros, Nucl. Phys. B **397**, 515 (1993); A. Pomarol and M. Quiros, Phys. Lett. B **438**, 255 (1998); I. Antoniadis, S. Dimopoulos, A. Pomarol and M. Quiros, Nucl. Phys. B **544**, 503 (1999); A. Delgado, A. Pomarol and M. Quiros, Phys. Rev. D **60**, 095008 (1999); R. Barbieri, L. J. Hall and Y. Nomura, Phys. Rev. D **63**, 105007 (2001), hep-ph/0106190 and hep-th/0107004; Y. Kawamura, hep-ph/0012125 and Prog. Theor. Phys. **105**, 691 (2001); G. Altarelli and F. Feruglio, Phys. Lett. B **511**, 257 (2001); A.B. Kobakhidze, hep-ph/0102323; N. Arkani-Hamed, L. Hall, Y. Nomura, D. Smith and N. Weiner, Nucl. Phys. B **605**, 81 (2001); L. Hall and Y. Nomura, hep-ph/0103125; Y. Nomura, D. Smith and N. Weiner, hep-ph/0104041. A. Hebecker and J. March-Russell, hep-ph/0106166.

[6] E. A. Mirabelli and M. E. Peskin, Phys. Rev. D **58**, 065002 (1998); M. Chaichian, A. B. Kobakhidze and M. Tsulaia, Phys. Lett. B **505**, 222 (2001); D. Marti and A. Pomarol, hep-th/0106256; A. Hebecker and J. March-Russell, hep-ph/0107039.

[7] C. Kounnas and B. Rostand, Nucl. Phys. B **341**, 641 (1990); E. Kiritsis and C. Kounnas, Nucl. Phys. B **503**, 117 (1997); I. Antoniadis, E. Dudas and A. Sagnotti, Nucl. Phys. B **544**, 469 (1999) and Phys. Lett. B **464**, 38 (1999); I. Antoniadis, G. D'Appollonio, E. Dudas and A. Sagnotti, Nucl. Phys. B **553**, 133 (1999); C. Angelantonj, I. Antoniadis, G. D'Appollonio, E. Dudas and A. Sagnotti, Nucl. Phys. B **572**, 36 (2000).

[8] J. Bagger, F. Feruglio and F. Zwirner, hep-th/0108010.

[9] P. Horava, Phys. Rev. D **54** (1996) 7561; A. Lukas, B. A. Ovrut and D. Waldram, Phys. Rev. D **57**, 7529 (1998). K. A. Meissner, H. P. Nilles and M. Olechowski, Nucl. Phys. B **561** (1999) 30.

[10] Y. Hosotani, Phys. Lett. B **126**, 309 (1983), Phys. Lett. B **129**, 193 (1983) and Annals Phys. **190**, 233 (1989).

[11] K. R. Dienes, E. Dudas and T. Gherghetta, Phys. Rev. D **62**, 105023 (2000).